7.7 ON THE BUDGET OF TURBULENCE KINETIC ENERGY IN THE UNSTABLE ATMOSPHERIC SURFACE LAYER

K.G. McNaughton*
School of GeoSciences, University of Edinburgh, Scotland

1. INTRODUCTION

In a recent paper McNaughton (2004) presented a new model for the structure of turbulence in the unstable atmospheric surface layer. This model describes turbulence in the ASL as a self-organizing system of coherent structures called 'TEAL' (Theodorsen ejection amplifier-like) structures. Each TEAL structure is initiated by a sharp updraft from near the ground, called an ejection. The oncoming sheared flow lifts over and curls inwards behind each ejection in a vortical motion about a hairpin-shaped core. This motion folds in on itself so that fluid within its arc converges and, with nowhere else to go, squirts outwards and backwards into the flow as a second, larger ejection. The individual energy and form of each TEAL structure depends on the local conditions in the flow where it develops, and only the best-formed TEAL structures produce ejections that are powerful and upright enough to initiate further, larger TEAL structures. An upscale sequence of TEAL structures constitutes a TEAL cascade. Wall-bounded shear turbulence consists of inter-acting, or 'competing' TEAL cascades where only the most symmetrical and powerful TEAL structures successfully initiate another cycle. TEAL cascades are initiated near the ground at small scale and develop upwards, driven by the local shear.

Empirical evidence, principally from the ***uw*** cospectrum, shows that the local structure of the turbulence is independent of stability throughout the unstable range. This is incompatible with Monin-Obukhov similarity theory. The purpose of this paper is to use the TEAL model to predict the budget of turbulence kinetic energy (TKE) in the unstable atmospheric surface layer, and to compare this with experimental results.

2. HAND-DOWN IN SCALE OF MOMENTUM

Before proceeding we must first establish some basic characteristics of the way momentum and TKE are transported by the turbulence. TEAL structures, being momentum - transporting structures, move faster air downwards and slower air upwards within their volume. This changes the distribution of shear within them, decreasing it in their upper parts and increasing it in their lower parts. Each TEAL structure is 'attached' to the ground and so has a footprint where it contacts the ground. Within such a footprint the shear is enhanced up to about half the height of the TEAL structure. This shear varies in direction, the flow having a radial component inwards towards the seat of the next ejection as well as a mean component in the direction of the larger TEAL structure. A whole cohort of new TEAL cascades grows within this footprint, each with power and direction reflecting the shear strength and direction at its particular position within the footprint. Thus momentum is handed down from each larger TEAL structure to a whole cohort of smaller TEAL structures growing within its footprint. The members of the new cohort do the same thing, handing the momentum on down to further cohorts of even smaller TEAL structures, and so on. Since the evolutionary time scale of each eddy is the inverse of the shear that drives it, each new cohort of TEAL structures has a faster cycle time than the last, so smaller TEAL structures can adapt continuously to changing conditions within footprints, even as the larger TEAL structures develop. The result is a fractal structure in the momentum transport process, with momentum being handed down to larger and larger numbers of smaller and smaller TEAL structures at each scale.

An essential feature of this process is that any pattern imposed on the system at large scale is transmitted down through the scales as a mean property of larger and larger numbers of smaller and smaller eddies. Thus a gust of wind is both the motion of a large eddy and the co-ordinated motions of many smaller eddies near the ground. In a convective outer layer the largest eddies scale on outer parameters of the flow and create large-scale variations in shear across the surface layer. This outer-scale pattern is then passed down through the cohorts of smaller and smaller TEAL structures towards the ground, where it causes low-frequency fluctuations in the shear stress. This is so even while all the momentum is transmitted by eddies whose individual development depends solely on the very local conditions in the flow and so scale on inner parameters of the flow.

This hand-down of momentum is accompanied by a hand-down of kinetic energy, so the ASL is powered partly by kinetic energy handed down from above and partly by energy produced by buoyancy acting within it.

Corresponding author address: K.G. McNaughton, School of GeoSciences, University of Edinburgh EH9 3JU;.e-mail keith.mcnaughton@ed.ac.uk

3. The TKE budget

Consider now the TKE budget of an unstable ASL beneath a convective outer layer. This budget can be written as

$$\overline{u'w'}\,\frac{\partial \overline{u}}{\partial z} + \frac{\partial \overline{w'e}}{\partial z} = -\frac{\partial \overline{w'p'}}{\partial z} + \frac{g}{T_0}\,\overline{w'\theta'} + \varepsilon \qquad (1)$$

where the symbols have their usual significances. Our proposition is that the structue of the turbulence in the unstable ASL is insensitive to buoyancy. That is, we propose that the statistics of the forms and numbers of the eddies, but not their velocity scale, are insensitive to instability. We can non-dimensionalize (1) by dividing through by u_*^3/kz, where u_* is friction velocity, k is von Karman's constant and z is observation height. Thus

$$0 = \phi_m - \frac{z}{L} + \phi_e + \phi_p + \phi_\varepsilon \qquad (2)$$

where the two TKE production terms on the left of (1) now have reversed signs and appear on the right of (2). In this form the terms are called shear production, buoyant production, turbulent transport, pressure transport and dissipation rate, respectively.

3.1 PRESSURE TRANSPORT

Though the structure of the turbulence is unaffected by buoyancy we must, nevertheless, accept that buoyancy forces will act on the turbulence, and that they will tend to accelerate warmer air parcels upwards and cooler ones downwards. If this has no effect on the structure of the turbulence then buoyancy forces must be opposed by equal and opposite pressure reaction forces, at least on average. That is to say, the production of gravitational potential energy must be matched by an equal production of pressure potential energy. The pressure transport term then equals the buoyant production term in (1) and (2).

This would be easy to imagine if the turbulence were to behave like a rigid machine, with its parts connected by gears and connecting rods. Fluids are not so rigid so we do not expect an exact balance of buoyancy and pressure reaction forces at each point in the flow. We require only that local changes in the flow brougt about by buoyancy be compensated in the whole flow by the overriding control exerted by the selection of TEAL structures able to initiate next stages in the TEAL cascades. Figure 1 shows that observed pressure transport and buoyant production terms are equal to within the accuracy of the published measurements, in agreement with our model.

3.2 TURBULENT TRANSPORT

The two terms on the left of (1) together represent the divergence in the flux of kinetic energy downwards from the outer part of the boundary layer. This is a negative quantity because kinetic energy is dissipated as heat within

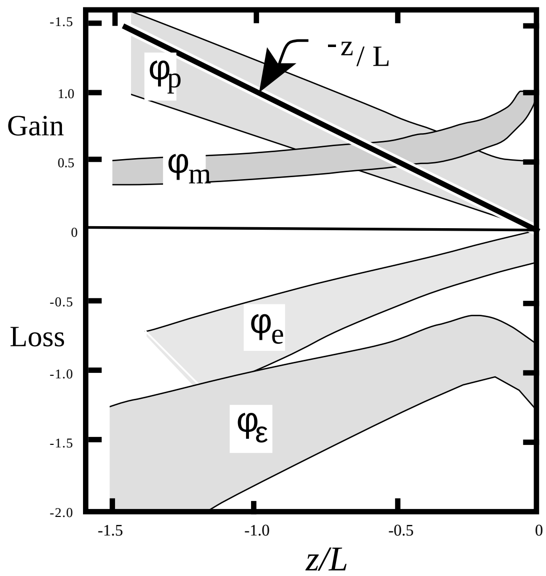

Figure 1. TKE budget for unstable surface layers from Högström (1990) as summarised by Wyngaard (1992). Experimental results fall within the bands indicated. Shear production is shown as a band rather than a line to more fairly represent the results summarized by Högström (1996).

the surface layer, with none being transferred directly to rigid land surfaces.

We can write the flux of kinetic energy as $\overline{w(u^2 + v^2 + w^2)}/2$. The terms on the left of (1) are obtained by decomposing the velocities in this expression into mean and fluctuating parts and setting $\overline{w}$ and $\overline{v}$ to zero, then taking the vertical divergence. The fluctuations indicated by the primes have two origins: one associated with the motions of the transporting eddies within the surface layer—the TEAL structures and their decay products; and the other associated with the variable forcing of the surface layer by the larger-scale eddies of the outer layer.

Almost all of the power in the spectrum of vertical velocity is contributed by eddies that develop within the surface layer and whose motions scale on the inner parameters, z, and u. That is to say, the outer-scale motions are almost horizontal at the top of the surface layer. These outer-scale eddies modulate the power and direction of the TEAL cascades, but make no net contribution to momentum transport. To handle this situation we extend the Reynolds decomposition of the velocity components to write

$$u = \overline{u} + \tilde{u} + u'$$

$$v = \overline{v} + \tilde{v} + v'$$

and

$$w = \overline{w} + \tilde{w} + w' \qquad (3)$$

where components with tildes correspond to outer-scale components while those bearing primes correspond to inner-scale components, as discussed by McNaughton and Laubach (1998). Here $\overline{w}$ and $\overline{v}$ are zero as before, as are the outer-scale fluctuations of ***w***, so $\tilde{w} = 0$. Momentum is transported entirely by inner-scale eddies, but the momentum flux itself has a mean value and displays an outer-scale pattern of variation as well as showing rapid, inner-scale fluctuations. We therefore write the streamwise and transverse components of the instantaneous momentum fluxes as the sum of mean, outer-scale and inner-scale parts:

$$u'w' = \overline{\tau}_x + \tilde{\tau}_x + \tau_x' \; ; \qquad v'w' = \tilde{\tau}_y + \tau_y' \qquad (4)$$

Using (3) and (4) and setting all cross-scale covariances to zero, we obtain

$$\overline{\tau}\,\frac{\partial \overline{u}}{\partial z} + \tilde{\tau}_x\,\frac{\partial \tilde{u}}{\partial z} + \tilde{\tau}_y\,\frac{\partial \tilde{v}}{\partial z} + \frac{\partial \overline{w'e}}{\partial z} \qquad (5)$$

as the left side of (1), where $\tilde{\tau}_x = \widetilde{u'w'}$, $\tilde{\tau}_y = \widetilde{v'w'}$ and ***e*** is TKE; $e = \overline{w\,(u'^2 + v'^2 + w^2)}/2$. The first and last terms are similar to terms in (1), but in (1) the outer-scale shear production terms are included within the turbulent transport term, while in (5) they are separated. The outer-scale modulation causes extra kinetic energy to be transported down into the surface layer, so dissipation must be enhanced within the surface layer.

3.3 THE WHOLE TKE BUDGET

We have already found that

$$\phi_p = -z/L \qquad (6)$$

The mean and variable shear production terms represent the divergence in the rate of work done by the atmosphere above ***z*** on the whole atmosphere below ***z***, so these terms represent real production of TKE. The last term in (5) has no simple physical interpretation but is an artefact caused by doing the double Reynolds decomposition about mean velocities with a non-logarithmic profile. Rather than evaluate it directly we evaluate the 'turbulent transport' term arising from the triple decomposition. In this we expect that total shear production (mean + variable) plus buoyant production must equal dissipation. This gives

$$\frac{\partial \overline{w'e}}{\partial z} = -\frac{\partial \overline{w'p'}}{\partial z} \qquad (7)$$

where the primes here indicate the inner-scale fluctuations of the triple decomposition.

To discuss the other terms of the TKE budget we need more information on the variable shear production. This involves modelling. We recall the relationship for the mean shear production

$$-\overline{\tau}\,\frac{\partial \overline{u}}{\partial z} = \frac{u_*^3}{kz}\,\phi_m \qquad (8)$$

and so write

$$-\overline{\tilde{\tau}_x\,\frac{\partial \tilde{u}}{\partial z}} - \overline{\tilde{\tau}_y\,\frac{\partial \tilde{v}}{\partial z}} = \frac{\overline{v_*^3\,\tilde{\phi}_m}}{k\,z} \qquad (9)$$

where $v_*^2 = \tilde{\tau}_x/\rho$ by analogy with u_*.

Since local turbulence structure is exactly the same regardless of whether a larger-scale pattern is superimposed or not, we expect $\tilde{\phi}_m = \phi_m$, so we write

$$-\overline{\tilde{\tau}_x\,\frac{\partial \tilde{u}}{\partial z}} - \overline{\tilde{\tau}_y\,\frac{\partial \tilde{v}}{\partial z}} = \frac{\overline{v_*^3}}{k\,z}\,\phi_m \qquad (10)$$

Equation (1) then becomes

$$\frac{1}{k\,z}\left(u_*^3 + \overline{v_*^3}\right)\phi_m + \frac{g}{T_0}\,\overline{w'\theta'} = \frac{1}{kz}\left(u_*^3 + \overline{v_*^3}\right) \qquad (11)$$

The dissipation term is parameterized this way because the structure of the turbulence is unchanged by instablity and the equation must balance also for a neutral surface layer disturbed by mechanically-generated outer turbulence, as migh happen in a valley with topographically-generated turbulence overhead. In effect, (11) is just (1) but with ($u_*^3 + \overline{v_*^3}$) replacing u_*^3 throughout. The velocity scale of the turbulence is different, but not its structure.

We now have the information we need to parameterize all the terms of the TKE budget in terms of the usual parameters plus $\overline{v_*^3}/u_*^3$. Non-dimensionalizing (11) using kz/u_*^3 gives the TKE budget as

$$\left(1 + \frac{\overline{v_*^3}}{u_*^3}\right)\phi_m - \frac{z}{L} = 1 + \frac{\overline{v_*^3}}{u_*^3} \qquad (12)$$

The term on the right is the dimensionless dissipation rate

$$\phi_\varepsilon = -\left(1 + \frac{\overline{v_*^3}}{u_*^3}\right) \qquad (13)$$

(Mean) shear production is given by

$$\phi_m = 1 + \frac{z}{L}\left(1 + \frac{\overline{v_*^3}}{u_*^3}\right)^{-1} \qquad (14)$$

and turbulent transport (traditional definition, including variable shear transport) by

$$\phi_e = \frac{z}{L} + \frac{\overline{v_*^3}}{u_*^3} \qquad (15)$$

These with (6) define all the terms of the dimensionless TKE budget (2).

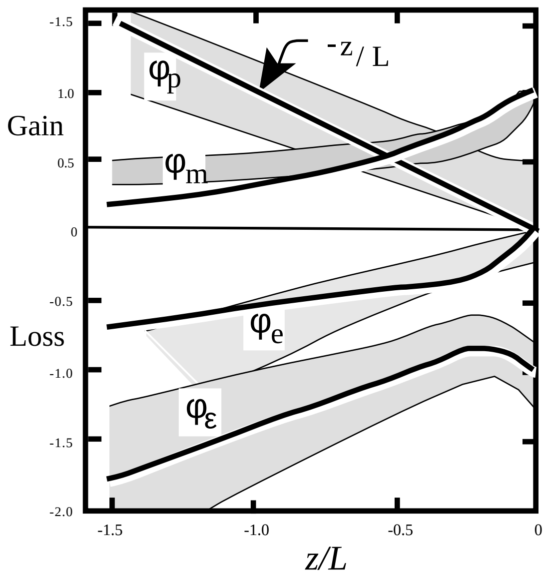

Figure 2. Model predictions for the terms of the TKE budget of the unstable atmospheric surface layer (solid lines) with experimental results (shaded bands) as in Fig. 1. The solid line drawn through the dissipation band is a fitted line, assumed to result from a statistical relationship between $\overline{v_*^3}/u_*^3$ and -z/L. The other lines are calculated from this relationship,

The relationships (13) - (15) depend on the scaling parameter $\overline{v_*^3}/u_*^3$, so they can not be plotted as single-valued functions of -z/L. This suggests that an inappropriate similarity model contributes greatly to the width of the bands in Figure 1. In order to compare this collection of experimental results with (13) - (15) we can assume that the band of results for the dissipation rate represents a broad statistical relationship between z/L and $\overline{v_*^3}/u_*^3$. Such a relationship is expected because both parameters depend on the surface heat flux. Thus we use the mid-line drawn through the experimental results to represent the relationship between z/L and $\overline{v_*^3}/u_*^3$, via (13). We then use this to calculate the other relationships using (14) and (15). The lines for buoyant production and pressure transport can be plotted directly against -z/L using (6). Results are shown in Figure 2. The comparisons do not provide an absolute test of the model, but they do test the model relationships are internally consistent. This consistence is generally very good.

4 CONCLUSIONS

The structural model of McNaughton (2004) leads to a TKE budget for the unstable atmospheric surface layer that is in good agreement with published results. The model proposes that the local structure of the turbulence is unaffected by instability, but is modulated by the larger-scale motions originating in the outer layer. Particular conclusions are:

- The pressure transport term in the TKE equation represents the conversion of gravitational potential energy to pressure potential energy without net production of motion, and so without altering the structure of the turbulence.
- TKE is transported down from the outer layer into the surface layer. This has both mean and variable components. The mean part, with its sign reversed, is usually called 'shear production' of TKE, while the variable part is usually included within the 'turbulent transport term'. As a result, the TKE budget is not closed when written in the traditional way.
- The extra TKE transported down into the surface layer affects transport processes but not mean momentum transfer, so u_* is not the correct velocity scale for turbulence processes in the atmospheric surface layer.

Overall. the results support the empirical principle of complete insensitivity of local turbulent structure to local instability (-z/L). Further empirical work is needed to define the limits of this principle.

5 REFERENCES

Högström, U., 1990: 'Analysis of turbulence structure in the surface layer with a modified similarity formulation for near neutral conditions', ***J. Atmos. Sci.***, **47**, 1949-1972.

Högström, U., 1996: 'Review of some basic characteristics of the atmospheric surface layer', ***Bound.-Layer Meteor.***, **78**, 215-246.

McNaughton, K. G., 2004: 'Turbulence structure of the unstable atmospheric surface layer and transition to the outer layer', ***Bound.-Layer Meteor.***, **112,** 199-221,

McNaughton, K. G., and Laubach, J., 1998: 'Unsteadiness as a cause of non-equality of eddy diffusivities for heat and vapour at the base of an advective inversion, ***Bound.-Layer Meteor.***, **88**, 479-504.

Wyngaard, J. C., 1992: 'Atmospheric turbulence', ***Ann. Rev. Fluid Mech.***, **24**, 205-233.