The rise and fall of Monin-Obukhov theory

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1 Introduction

The Monin and Obukhov similarity theory (Monin and Obukhov, 1954) and the Deardorff (1970) convective scaling theory have together provided conceptual and practical foundations for almost all modeling of the unstable atmospheric boundary layer for the past four decades. Neither theory was scientifically conclusive but any doubts were swept aside by the results of two great field experiments: the 1967 'Kansas' experiment (Haugen et al., 1971) and the 1973 'Minnesota' experiment (Kaimal et al, 1976). The experimenters themselves were so impressed by the support they provided for the Monin-Obukhov similarity theory that Kaimal (1973) was able to claim that "*with proper nondimensionalization, all spectra and cospectra in the surface layer can be reduced to a set of universal curves*". Most scientists have agreed with him since that time.

The problem is that Kaimal was overly optimistic, confusing good progress in bringing empirical order to wind and scalar profiles, spectra and other turbulence statistics, with a final solution to that problem. The purpose of this article is to point out some of the problems with Monin-Obukhov theory and to provide a brief introduction to an alternative approach for convective boundary layers (CBLs).

2 The Monin-Obukhov similarity model

Most text books present the Monin-Obukhov similarity theory as a dimensional argument based on the set of governing parameters, z, u_* and gq/T, where symbols have their usual meanings and q is heat flux at the ground, but it is more than that. The significance of the Obukhov length L had already been deduced by Obukhov in 1943, though not published until after the world war, in 1946 (Obukhov, 1971). But Obukhov's work was based directly on the mixing-length model of Prandtl, so its conclusions were linked to the notions of gradient diffusion and a mixing length that varied with height. Monin and Obukhov tried to place this theory on a more general foundation, and to point out its applications.

Their method of similarity analysis was

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based on finding the conditions for similarity of the differential equations that express local relationships at any point in the fluid, plus u_* and q, which described the momentum and heat flux boundary conditions at the ground. Monin and Obukhov (1954) then introduced the concept of an atmospheric surface layer, defined as the layer where momentum and heat fluxes are sufficiently constant with height that the surface fluxes can be taken equal to the local fluxes at height z. By this device u_* and q could be considered local variables within the surface layer. Monin and Obukhov also introduced height z as a parameter, explaining that "*it is natural to assume that changes in mean velocity and temperature with height can be expressed by coordinate z*". This is a different kind of assumption since z expresses the difference between the local observation height and somewhere else (the ground), so z is not a local parameter. Monin and Obukhov (1954) had introduced z earlier in their paper, in a discussion of a neutral wind profile whose properties were self-similar with height. Though we might agree that z should be a parameter, its addition fits uncomfortably with the 'local' arguments used to choose the other parameters.

This difficulty with z comes to a head when Monin and Obukhov consider the asymptotic approach to windless (free) convection. Then u_* disappears from the list of parameters, and their methods identify only two parameters: g/T and q. They write "*We cannot form a length scale from the parameters q and g/T; therefore, the regime of purely thermal turbulence is "self-patterning"*". What they mean is that, in free convection, the turbulence organizes itself into patterns of motion (coherent structures perhaps) whose size depends on height above ground. One must admire their insight in coming to this conclusion because, until then, coherent structures had received very little attention in fluid dynamics. Had they gone further in this line of thinking they might have recognized that the self-similarity of neutral wind profiles is also an expression of the self-patterning nature of turbulence. Self-patterning occurs in real flows, and in well-formulated computer simulations, and so is an integral rather than a local property of the flow. If we admit z as a parameter, we may ask why other integral properties should not also be considered? Indeed, we might argue that <u>only</u> integral properties should be considered, since these are the ones that can be deduced experimentally from observations of real flows. This brings into question the whole selection process used by Monin and Obukhov, including whether purely local properties, such as q and u_*, should be used at all.

3 The Deardorff similarity model

Early on it was recognized that Monin-Obukhov similarity does not apply to horizontal velocity spectra, since the length scale of their peaks is independent of z (e.g. Bush and Panofsky, 1968). Kaimal (1978) argued that this could be explained by the effects of large eddies that obey mixed-layer scaling. A scaling theory for the large eddies was provided by Deardorff (1970), who was one of the first to perform large-eddy simulations of convection in the CBL. From his computations Deardorff found that the large coherent structures in a CBL can be scaled using the height of the capping inversion, z_i and the convective velocity scale $w_* = (z_i\, q\; g/T)^{1/3}$. The latter scale was compatible with Monin-Obukhov theory since the vertical velocity scale for the local free convection layer could be matched with the Deardorff scale simply by letting $z \to z_i$. This gave a consistent similarity model for the lower CBL, up to the bottom of the entrainment layer (Kaimal, 1976; Panofsky, 1977). The original Monin-Obukhov formulation still applied to vertical motions.

We cannot criticize Deardorff for neglecting integral properties of the CBL, since his results are based on results from integrations of the flow equations, but we can have concerns about the boundary conditions used in those integrations. Deardorff (1972) maintained his flow against surface drag by adjusting the horizontal pressure gradient at each time step. This adds energy to the mean flow throughout the CBL and so maintains the shear stress near the ground, but it contributes nothing to the energy of the large eddies which must, therefore, be driven by buoyancy alone. In real CBLs the flow is maintained principally by momentum entrained through the capping inversion (Stevens et al., 2002). If the entrainment velocity is $w_e[= dz_i/dt - s$, where s the subsidence velocity] then the momentum flux into the CBL is $w_e u_g$ (neglecting wind turning), where u_g is the geostrophic wind above the CBL. This momentum flux is associated with a flux of kinetic energy down into the boundary layer, $w_e u_g^2/2$, of which $w_e u_m^2/2$ goes to maintain the mean flow, u_m, while the remainder, $w_e(u_g^2 - u_m^2)/2$, goes to maintain the large-scale turbulence. That is to say, the energy of the large eddies derives from entrained kinetic energy as well as buoyant production within the CBL, while the Deardorff model has only buoyancy.

4 Tests of the Monin-Obukhov/Deardorff similarity model

Despite these theoretical concerns about MO/D theory, the real test is whether the theory can reduce experimental data to universal constants or relationships. In this the MO/D scheme can work reasonably well, if not perfectly. For example, wind and temperature profiles from a range of careful experiments show generally similar shapes when scaled according to the Monin-Oukhov scheme, though the dimensionless gradients spread over a range of about 30% for given $-z/L$ (Högström, 1996). Statistics like the scaled temperature variance, σ_T/T_*, from different experiments also display generally similar relationships with z/L in unstable conditions, but scatter about the trend is typically large at about 50% (Kader and Yaglom, 1990, Fig 4), and the means differ between experiments. Other statistics and functions behave similarly, failing to collapse exactly onto well-defined universal values and functions.

These limitations are well known, and most researchers simply accept that the scatter reflects the differences between real conditions encountered during field experiments and the ideal conditions of perfect atmospheric steadiness and land homogeneity required by the theory. Perhaps so, but this argument would be more convincing if there were studies to show how unsteadiness and inhomogeneities affect compliance. Instead, these arguments often simply provide an excuse for bad performance whenever results are not as expected.

More demanding tests have been used. For example, one can look at higher-order moments (e.g. Högström, 1990), or for systematic relationships between the residuals from a MO/D analysis and other parameters (Johansson et al, 2002). The results are difficult to interpret, but generally weaken our confidence in the MO/D theory. Other results suggest that some of the universal curves of MO/D theory are not universal at all. Thus Smedman et al (2007), in near-neutral conditions over land and sea, observed temperature spectra that display two peaks: one at a wavelength normally associated with the z_i-scale eddies that fill the CBL, and the other where one would expect to see small-scale shear processes. Heat flux cospectra with two peaks present a similar problem. Most damaging of all are the results of McNaughton et al. (2007) for very unstable conditions: these show that the position of the peak of the temperature spectrum depends on the mixed length scale $z_i^{1/2} z^{1/2}$, which is not a possibility with the MO/D model.

The Deardorff scaling theory may also be tested. That z_i is the height scale seems to be correct, but there have been no direct experimental tests to show that the velocity scale of these eddies is w_*. One test is that the energy of the large eddies ($\propto w_*^2$) should be constant when scaled with $z_i gq/T$. Writing the former as $z_i \epsilon_o$, where ϵ_o is the dissipation rate in the bulk of the CBL, this test is that $gq/T\epsilon_o$ should be constant. Results from the literature show $gq/T\epsilon_o$ to vary from 0.4 to more than 1.7 (see summary in Laubach and McNaughton, 2009).

5 A new approach to similarity modeling

There seems to be no alternative to similarity modeling. Fortunately, atmospheric flows, with their very high Reynolds and Rayleigh numbers, seem particularly well suited to this approach. The key is to find the right set of basic variables and the right way to use them. This involves choosing the right conceptual model.

A new conceptual model has been used by Laubach and McNaughton (2009, and references therein). This model views the CBL as a complex dynamical system in which the turbulence at all scales forms a single, interconnected system. The basic parameter set is not very different to the MO/D parameter set. Where the MO/D model has $\{u_*, z, gq/T, z_i\}$, with $L = -ku_*^3/gq$ as an important derived parameter, the new model has $\{u_\epsilon, z, \epsilon_o, z_i\}$, with $z_s = ku_\epsilon^3/\epsilon_o$ as an important derived parameter. Here u_ϵ is related to the energy produced by the shearing action of the wind rather than to the momentum transfer, and the energy dissipation rate in the bulk CBL, ϵ_o, replaces the narrower buoyant production gq/T of the MO/D model.

With such a similar parameters set, one might expect the similarity relationships to be pretty much the same, but this is not so. Unlike MO/D theory, where scaling is applied to the whole flow, the new model applies empirical scaling analysis separately to the different kinds and sizes of eddies. The most remarkable outcome is that eddies and heat plumes can have mixed length, energy and velocity scales. Mixed scales are the geometric mean of the scales that arise when different kinds of eddies interact. For example, the position of the peaks of temperature spectra in the surface friction layers occurs where $\kappa z_i^{1/2} z_s^{1/2} = 4.5$ and κ is wavenumber. A variety of other mixed scales are found, with different mixed scales applying within the surface friction layer and above it.

The new scheme explains many of the anomalies of MO/D theory. However, it is not a local theory so the flow cannot be described in terms of experimental parameters measured at just one point. Remote sensing instruments, such as sodar, radar or lidar, will be needed to supplement the sonic anemometers etc. of a typical experiment. The first round of such experiments will be to verify the new model, and that will be a major job in itself.

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